Schwind ATSS

0

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-3760

ST - SP - PP - 10 234

NASA TT F-8985

FACILITY FORM 602

N65 11303 (ACCESSION NUMBER)

(THRU)

8 (PAGES)

1 (CODE)

(NASA CR OR TMX OR AD NUMBER)

29 (CATEGORY)

MAGNETIC FIELD IN A FLUID OF INFINITE CONDUCTIVITY PLACED IN A UNIFORM GRAVITATIONAL FIELD

by

Donald McLean

[FRANCE]

OTS PRICE

XEROX $ 1.00 FS

MICROFILM $ 0.50 mf

6 NOVEMBER 1964

NASA TT F-8985
ST - SP - PP - 10234

MAGNETIC FIELD IN A FLUID OF INFINITE CONDUCTIVITY PLACED IN A UNIFORM GRAVITATIONAL FIELD

Annales d'Astrophysique
Tom 27, No. 4, pp. 195-197,
Paris, 1964

by Donald McLean

ABSTRACT

11303

This paper describes a simple class of equilibrium configurations of a magnetic field in a fluid of infinite conductivity which is subject to a uniform gravitational force. This class is typified by a tube of force, surrounded by a region of zero magnetic field and supported by the pressure gradient in the fluid. The form of such a tube is shown by the shaded region of Figure 2.

Author

*

* *

*

The purpose of this communication is to present calculations relative to a class of configurations of a magnétic field, which can, for example, exist in the equilibrium in the high solar corona. The calculations take into account the gravitational force, assumed uniform, but one supposes that the conductance of the corona plasma is infinite and the temperature is uniform everywhere.

---

* LE CHAMP MAGNÉTIQUE DANS UN FLUIDE DE CONDUCTIVITÉ INFINIE PLACÉ DANS UN CHAMP GRAVITATIONNEL UNIFORME

The shape of the magnetic field described resembles a tube of force whose extremities are fixed in the photosphere, and which are supported in the corona by the gas pressure. The magnetic field can be zero at the exterior of this tube. Since the magnetic forces of this model are compensated by the gas pressure, it is evident that the intensity of the magnetic field is strictly limited, of the order of one gauss for the corona. On the other hand, this model can only have interest in the regions which are devoid of the magnetic field, or nearly so, and this because the effect of the gas pressure is here supposed predominant.

In order that the isothermic gas and the infinite conductivity be at equilibrium in the field of uniform gravity g, it is necessary that the equation (1) be satisfied:

$$0 = \frac{\partial}{\partial \boldsymbol{r}} p + \rho \boldsymbol{g} + \boldsymbol{j} \wedge \mathbf{B}. \tag{1}$$

In this equation

$\frac{\partial}{\partial \boldsymbol{r}}$ is the operator gradient,

p is the gas pressure, isotropic at point r.

ρ is the density at the same point.

g is the acceleration due to gravity.

j is the electric current at point r,

and $B = \mu_0 H$ is the magnetic vector in this point.

(μ_0 is the permittivity of free space).

We assume also that:

$$p = n\mathrm{KT} \qquad \text{and} \qquad \rho = mn,$$

whence

$$\rho = \frac{m}{\mathrm{KT}} p,$$

(n is the number of particles by unit volume in r and m is the average mass of a particle, which does not depend on r. T is the temperature (also uniform) and K is Boltzmann constant).

With the definition:

$$\alpha = \frac{m}{KT} g, \tag{2}$$

and taking into account the Maxwellian equation, which becomes:

$$j = \frac{\partial}{\partial r} \wedge \mathbf{H},$$

one can put equation (1) under the following form:

$$0 = \left(\frac{\partial}{\partial r} + \alpha\right) p + \frac{1}{\mu_0}\left(\frac{\partial}{\partial r} \wedge \mathbf{B}\right) \wedge \mathbf{B}.$$

If one defines the quantity A (r) by:

$$p = \frac{A}{2\,|\alpha|\,\mu_0}\, e^{-\alpha . r},$$

and if, moreover, one takes $\frac{1}{2|\alpha|}$ as the unit of length, one easily has:

$$\frac{\partial A}{\partial r} = -\, e^{-2\hat{\alpha} . r} \operatorname{rot} \mathbf{B} \wedge \mathbf{B}, \tag{3}$$

whence

$$\hat{\alpha} = \alpha/|\alpha|.$$

If r is expressed in the system of cartesian coordinates r = (x,y,z) where the z axis is parallel to the vector $\hat{\alpha}$, one can define the new coordinates $\rho = (\xi, \eta, \zeta)$ such that:

$$\begin{aligned} \xi &= \cos x \, e^{-z} & x &= \operatorname{arctg} \zeta/\xi \\ \eta &= y & y &= \eta \\ \zeta &= \sin x \, e^{-z} & z &= -\frac{1}{2} \ln (\xi^2 + \zeta^2). \end{aligned}$$

Equation (3) can be written according to three components, of which one among them is:

$$\frac{1}{h\xi}\frac{\partial}{\partial \xi} A = e^{-2z(\rho)} \Big\{ B_\zeta \frac{1}{h_\zeta h_\xi}\left(\frac{\partial}{\partial \zeta} h_\xi B_\xi - \frac{\partial}{\partial \xi} h_\zeta B_\zeta\right) - B_\eta \frac{1}{h_\xi h_\eta}\left(\frac{\partial}{\partial \xi} h_\eta B_\eta - \frac{\partial}{\partial \eta} h_\xi B_\xi\right) \Big\} \tag{4}$$

The two other components can be derived from equation (4) by two cyclical permutations of ξ, η, et ζ.

In these equations,

$$h'_\eta = 1 \; h_\xi = h_\zeta = \sqrt{\xi^2 + \zeta^2} = e^{-z} = h.$$

For the special case, in which we are interested, where $B_\psi \equiv B_\eta \equiv 0$, equation (4) becomes:

$$\frac{\partial}{\partial \rho} A = \left(\frac{\partial}{\partial \rho} \wedge \mathbf{B}'\right) \wedge \mathbf{B}' \tag{5}$$

where

$$\mathbf{B}' = (hB_\xi, 0, hB_\zeta).$$

One verifies easily that the necessary condition $\frac{\partial}{\partial r} . \mathbf{B} = 0$ is satisfied if

$$\frac{\partial}{\partial \rho} \cdot \mathbf{B}' = 0 \tag{6}$$

(always to conditions that B_η be zero).

One sees immediately that vector B' satisfies the same equations as the magnetic field in the absence of a gravitational force. Nevertheless, the apparent generality of this result is strictly modified by the condition $B'_\eta = 0$.

It seems that there exist only two types of B' configurations which can satisfy equations (5) and (6). In one, already studied by Dungey [1] utilizing the same system of coordinates, the lines of force are concentric circles in the plane $\xi - \zeta$ and none of the quantities varies as a function of η.

The magnetic intensity can be chosen arbitrarily for each line of force, and then the quantity A which, to the approximation of a constant, represents the gas pressure, must be chosen of the fashion to satisfy equation (5).

In the other type of configuration, the lines of magnetic force are straight and parallel. This time, the magnetic field and the pressure can be functions of η, also the magnetic field is arbitrary, not only on each line of force of the plane, but equally for each plane. The variations of the pressure must always compensate for the variations of the magnetic intensity.

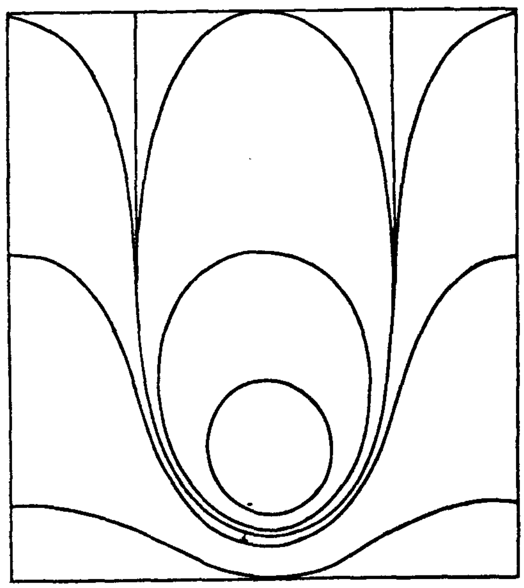

Figure 1

Configuration of the lines of force of the magnetic field for the case studied by Dungey.

In the original coordinates (x, y, z), the lines of force take the form represented in Figures 1 and 2. For Figure 1, as Dungey has already demonstrated, the lines of force are given by different B values in the expression:

$$e^{-2z} - 2\,e^{-z}\cos x + 1 = B.$$

The form of lines in the Figure (2) is given by :

$$\cos x\, e^{-z} = C,$$

for different values of C.

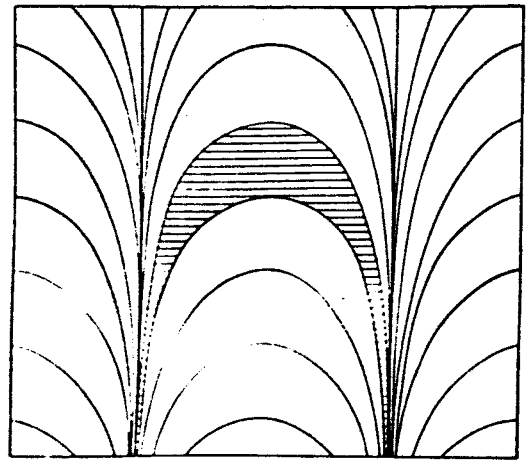

Figure 2

Configuration of lines of force of the magnetic field for the case studied in this article; as an example, the field can be zero outside the tube indicated by the shaded region.

Contrary to the case studied by Dungey, the one that is represented in Figure 2 can well satisfy the conditions at reasonable limits without modifications. It suffices to add a horizontal trait to Figure 2, which represents the photosphere (for example) and to consider the case where the field is zero outside a tube defined by a surface composed of certain lines of Figure 2. The shaded region of this figure represents a possible tube. One then has the image of a flux which emerges from the photosphere in a region and reenters in another. As to the corona, above the photosphere, the field is completely bounded in all directions (contrary to the case of Dungey which extends to infinity parallel to the y axis) and thus physically acceptable. The problems of stability and effects of dissipation remain to be solved. Nevertheless, intuitively, it seems that this field must be mechanically stable; in this regard the analogy with a heavy chain suspended by two extremities can be invoked. The effect of thermal perturbations is less evident.

This model has been presented in the hope that it would have some applications in the description of the dark regions of the corona, or more probable in the description of emission regions of metric waves by the synchrotron mechanism. Very likely, the remarkable resemblance of the form of this model with the quiescent solar prominence arches can only be accidental.

*** THE END ***

Manuscript received on
15 March 1964

REFERENCES

1. DUNGEY, J. W. M. N. 113, 180, 1953.

Contract No. NAS-5-3760
at Consultants and Designers, Inc.
Arlington, Virginia

Translated by
DIANE A. CHRZANOWSKI
Revised by
DR. ANDRE L. BRICHANT

DISTRIBUTION

GODDARD SFC

600	TOWNSEND	
610	MEREDITH	
611	McDONALD	[3]
612	HEPPNER	[3]
	SUGIURA	
	NESS	
613	KUPPERIAN	
615	BOURDEAU	
	SERBU	
614	LINDSAY	[3]
	WHITE	
640	HESS	[5]
660	GI for SS	[5]
252	LIBRARY	[5]
256	FREAS	

NASA HQS

SS	NEWELL, CLARK
SP	STANSELL
SG	NAUGLE
	SCHARDT
	ROMAN
	SMITH
	DUBIN
SL	LIDDEL
	BRUNK
	FELLOWS
	HIPSHER-HOROWITZ
SM	FOSTER
	ALLENBY
	GILL
RR	KURZWEG
RRP	GESSOW
RTR	NEILL
ATSS	SCHWIND
AO-4	ROBBINS

OTHER CENTERS

	AMES	
	SONETT	[5]
	94035	[3]
	LANGLEY	
160	ADAMSON, HESS	
213	KATZOFF, DAVIS	
235	SEATON	
185	WEATHERWAX	[2]
	UCLA	
	COLEMAN	
	UC BERKELEY	
	WILCOX	